International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 6, Nov 2014, 39-44

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

QUALITATIVE PROPERTIES OF SOLUTION OF CROSS-DIFFUSION MODEL OF KOLMOGOROV-FISHER TYPE BIOLOGICAL POPULATION TASK

D. K. MUHAMEDIYEVA

Junior Researcher, Centre for the Development of Software and Hardwre-Program Complexes,
Tashkent, Uzbekistan

ABSTRACT

In present work explores the issues of global solvability of the biological population task of Kolmogorov-Fisher type and qualitative properties of the solution of task on the basis of self-similar analysis. Considered a parabolic system of two quasilinear equations of reaction-diffusion. Suggested suitable initial approximation for fast convergence iterative process. Carried out numerical experiments with visualization for different values of system parameters. Modeling of growth processes of dissipative structures in reaction-diffusion (RD) systems contributes to the development of theoretical ideas about colonial organization of populations.

KEYWORDS: Cross-Diffusion, Biological Population, Parabolic System of Quasilinear Equations, Initial Approximation, Numerical Solution of the Iterative Process, Self-Similar Solutions

1. INTRODUCTION

Let's consider in the domain Q= {(t,x): 0< t < ∞, $x\in R^2$} parabolic system of two quasilinear reaction-diffusion equations of Kolmogorov-Fisher type biological population task

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{11}u_1^m + a_{12}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{11}u_1^m + b_{12}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_1(t)u_1(1-u_2^{\beta_1}) \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{21}u_1^m + a_{22}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{21}u_1^m + b_{22}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_2(t)u_2(1-u_1^{\beta_2}) \end{cases} \quad (1)$$

a_{ij}, b_{ij} - positive real numbers, $\beta_1, \beta_2 \ge 0$, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - desired solutions.

At $a_{ij} \ne 0$, $b_{ij} = 0$ or $a_{ij} = 0$, $b_{ij} \ne 0$ mathematical model (1) is a system of reaction-diffusion with diffusion coefficients $a_{ij}u_i^m \ge 0, b_{ij}u_i^m \ge 0$. In the case when at least one of the coefficients $a_{ij} \ne 0$ и $b_{ij} \ne 0$ (sign can be any), system (1) is the cross-diffusion (mutual-diffusion for i,j=1,2) [1-12].

Equation (1) is a generalization of the simple diffusion model for the logistic model of Malthusian type population growth [13-16] ($f_1(u_1,u_2) = u_1$, $f_1(u_1,u_2) = u_2$, $f_2(u_1,u_2) = u_1$, $f_2(u_1,u_2) = u_2$), Ferhulst type ($f_1(u_1,u_2) = u_1(1-u_2)$, $f_1(u_1,u_2) = u_2(1-u_1)$, $f_2(u_1,u_2) = u_1(1-u_2)$, $f_2(u_1,u_2) = u_2(1-u_1)$), and Allee type ($f_1(u_1,u_2) = u_1(1-u_2^{\beta_1})$, $f_1(u_1,u_2) = u_2(1-u_1^{\beta_2})$, $f_2(u_1,u_2) = u_1(1-u_2^{\beta_1})$, $f_2(u_1,u_2) = u_2(1-u_1^{\beta_2})$, $\beta_1 > 1,\ \beta_2 > 1$) for the case of dual nonlinear diffusion. When $\beta_1 \ge 1,\ \beta_2 \ge 1$, it can also be considered as the equation of nonlinear filtering, thermal conductivity with simultaneous exposure of the source and the absorption capacity of which is equal respectively $u_1, -u_2^{\beta_1}, u_2, -u_1^{\beta_2}$, under the influence of convective transfer with speeds $\frac{\partial u_1}{\partial x}$ and $\frac{\partial u_2}{\partial x}$ in a two-component nonlinear media [15,19].

Let’s consider a spatial analogue of the system of Volterra-Lotka with nonlinear power dependence of diffusion coefficient on population density. In the simplest case walterovich competitive interactions between populations can be constructed numerically, and in some cases analytically heterogeneous in space solutions [19]. In dissertation work "Cross diffusion systems" Toan Trong Nguyen, M.S. 2006 reviewed some qualitative properties of solutions of problem (1), but the solution is not obtained. The aim of this work is the investigation of qualitative properties of solutions of problem (1) based on the self-similar analysis and numerical solutions using modern computer technologies, research methods linearization to the convergence of iterative process with further visualization. Researched global solvability of problem (1) based on the self-similar approach.

Let's construct a similar system of equations (1) - easier to research the system of equations. Construct it by using nonlinear splitting [15], which will make the following substitution in (1)

$$u_1(t,x) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),x),$$

$$u_2(t,x) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),x),$$

leading (1) to the form

$$\begin{cases} \frac{\partial v_1}{\partial \tau_1} = \frac{\partial}{\partial x}\left[(a_{11}v_1^m + a_{12}v_2^m)\frac{\partial v_1}{\partial x} + (b_{11}v_1^m + b_{12}v_2^m)\frac{\partial v_2}{\partial x}\right] + k_1(t)\cdot e^{(\beta_1 k_2 - km)t} v_1 v_2^{\beta_1} \\ \frac{\partial v_2}{\partial \tau_2} = \frac{\partial}{\partial x}\left[(a_{21}v_1^m + a_{22}v_2^m)\frac{\partial v_1}{\partial x} + (b_{21}v_1^m + b_{22}v_2^m)\frac{\partial v_2}{\partial x}\right] + k_2(t)\cdot e^{(\beta_2 k_1 - km)t} v_1^{\beta_2} v_2 \end{cases} \quad (2)$$

$$v_1\big|_{t=0} = v_{10}(x), \quad v_2\big|_{t=0} = v_{20}(x).$$

By selecting $\tau = \tau_1 = \tau_2 = \frac{e^{kmt}}{km}$;, we get the following system of equations

$$\begin{cases} \frac{\partial v_1}{\partial \tau} = \frac{\partial}{\partial x}\left[(a_{11}v_1^m + a_{12}v_2^m)\frac{\partial v_1}{\partial x} + (b_{11}v_1^m + b_{12}v_2^m)\frac{\partial v_2}{\partial x}\right] + a_1(t)\tau^{b_1} v_1 v_2^{\beta_1}, \\ \frac{\partial v_2}{\partial \tau} = \frac{\partial}{\partial x}\left[(a_{21}v_1^m + a_{22}v_2^m)\frac{\partial v_1}{\partial x} + (b_{21}v_1^m + b_{22}v_2^m)\frac{\partial v_2}{\partial x}\right] + a_2(t)\tau^{b_2} v_1^{\beta_2} v_2, \end{cases} \quad (3)$$

Where $b_1 = \frac{(\beta_1 - \mathrm{m})}{m}$; $a_1 = k_1(t)\cdot[km]^{b_1}$; $b_2 = \frac{(\beta_2 - \mathrm{m})}{m}$; $a_2 = k_2(t)\cdot[km]^{b_2}$

If $b_i = 0$, and $a_i(t) = const$, $i = 1,2$, the system has the form

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial x}\left[(a_{11}v_1^m + a_{12}v_2^m)\dfrac{\partial v_1}{\partial x} + (b_{11}v_1^m + b_{12}v_2^m)\dfrac{\partial v_2}{\partial x}\right] + a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial x}\left[(a_{21}v_1^m + a_{22}v_2^m)\dfrac{\partial v_1}{\partial x} + (b_{21}v_1^m + b_{22}v_2^m)\dfrac{\partial v_2}{\partial x}\right] + a_2 v_1^{\beta_2} v_2. \end{cases} \quad (4)$$

In this case, on the basis of the method of splitting [19] are looking for a solution in the form

$$v_1(t,x) = \bar{v}_1(\tau) w_1(\tau(t),x), w_1(\tau(t),x) = f_1(\xi)$$

$$v_2(t,x) = \bar{v}_2(\tau) w_2(\tau(t),x) \quad , w_2(\tau(t),x) = f_2(\xi) , \quad \xi = \frac{x}{\sqrt{\tau_1}}$$

Where

$$\bar{v}_1(\tau) = (T_0+\tau)^{-\gamma_1}, \ \bar{v}_2(\tau) = (T_0+\tau)^{-\gamma_2}, \ T_0 > 0, \gamma_1 = \frac{1}{\beta_1}, \ \gamma_2 = \frac{1}{\beta_2},$$

And in the case $b_i \neq 0$, and $a_i(t) = const$, $i = 1,2$

$$\bar{v}_1(\tau) = (T_0+\tau)^{-\gamma_1}, \ \bar{v}_2(\tau) = (T_0+\tau)^{-\gamma_2}, \ T_0 > 0,$$

Where

$$\gamma_1 = \frac{b_2+1}{\beta_2}; \ \ \gamma_2 = \frac{b_1+1}{\beta_1}.$$

Then we then obtain a self-similar system

$$\begin{cases} \dfrac{d}{d\xi}\left[(a_{11}f_1^m + a_{12}f_2^m)\dfrac{df_1}{d\xi} + (b_{11}f_1^m + b_{12}f_2^m)\dfrac{df_2}{d\xi}\right] + \dfrac{\xi}{2}\dfrac{df_1}{d\xi} + \psi_1 f_1(1 - f_2^{\beta_1}) = 0, \\ \dfrac{d}{d\xi}\left[(a_{21}f_1^m + a_{22}f_2^m)\dfrac{df_1}{d\xi} + (b_{21}f_1^m + b_{22}f_2^m)\dfrac{df_2}{d\xi}\right] + \dfrac{\xi}{2}\dfrac{df_2}{d\xi} + \psi_2 f_2(1 - f_1^{\beta_2}) = 0, \end{cases} \quad (5)$$

And $\tau_1 = \tau_1(t)$ is chosen so

$$\tau_1(\tau) = \begin{cases} \dfrac{(T+\tau)^{-\gamma_1 m+1}}{-\gamma_1 m+1}, & if \ -\gamma_1 m + 1 \neq 0, \\ \ln(T+\tau), & if \ -\gamma_1 m + 1 = 0, \\ (T+\tau), & if \ m = 0, \end{cases}$$

If $\gamma_2 m_1 = \gamma_1 m_2$

System (5) has an approximate solution in the form

$$\bar{f}_1 = A(a - b\xi^2)_+^{\eta_1}, \ \bar{f}_2 = B(a - b\xi^2)_+^{\eta_2} \ \ (y)_+ = \max(0, y).$$

Investigation of qualitative properties of system (1) is allowed to perform a numerical experiment based on the values included in the system of numeric parameters. For this purpose, as the initial approximation was used asymptotic solutions. For the numerical solution of the problem for the linearization of system (1) has been used linearization methods of Newton and Picard. To construct a similar system of equations biological populations used method for nonlinear splitting [16, 19]

On the basis of research numerical experiments in

Below are the results of numerical experiments for different parameter values obtained in Mathcad.

Parameter values	$t=1$ $a_{11}=1, a_{12}=1, b_{11}=1, b_{12}=1$ $a_{21}=1, a_{22}=1, b_{21}=1, b_{22}=1$	$t=40$ $a_{11}=1, a_{12}=1, b_{11}=1, b_{12}=1$ $a_{21}=1, a_{22}=1, b_{21}=1, b_{22}=1$	$t=1$ $a_{11}=0, a_{12}=1, b_{11}=0, b_{12}=0$ $a_{21}=0, a_{22}=0, b_{21}=1, b_{22}=0$	$t=40$ $a_{11}=0, a_{12}=1, b_{11}=0, b_{12}=0$ $a_{21}=0, a_{22}=0, b_{21}=1, b_{22}=0$	$t=40$ $a_{11}=0, a_{12}=0, b_{11}=0, b_{12}=1$ $a_{21}=1, a_{22}=0, b_{21}=0, b_{22}=0$
$m_1=0.4$, $m_2=($ $\beta_1=5,\ k_1=2$ $\beta_2=7,\ k_2=5$ $eps=10^{-3}$					
$m_1=1.7$, $m_2=1.9$ $\beta_1=5,\ k_1=2$ $\beta_2=7,\ k_2=5$ $eps=10^{-3}$					
$m_1=2.7$, $m_2=2.9$ $\beta_1=5,\ k_1=2$ $\beta_2=7,\ k_2=5$ $eps=10^{-3}$					
$m_1=0.4$, $m_2=0.7$ $\beta_1=0.5,\ k_1=2$ $\beta_2=0.7,\ k_2=3$ $eps=10^{-3}$					
$m_1=1.4$, $m_2=1.4$ $\beta_1=0.5,\ k_1=2$ $\beta_2=0.7,\ k_2=5$ $eps=10^{-3}$					

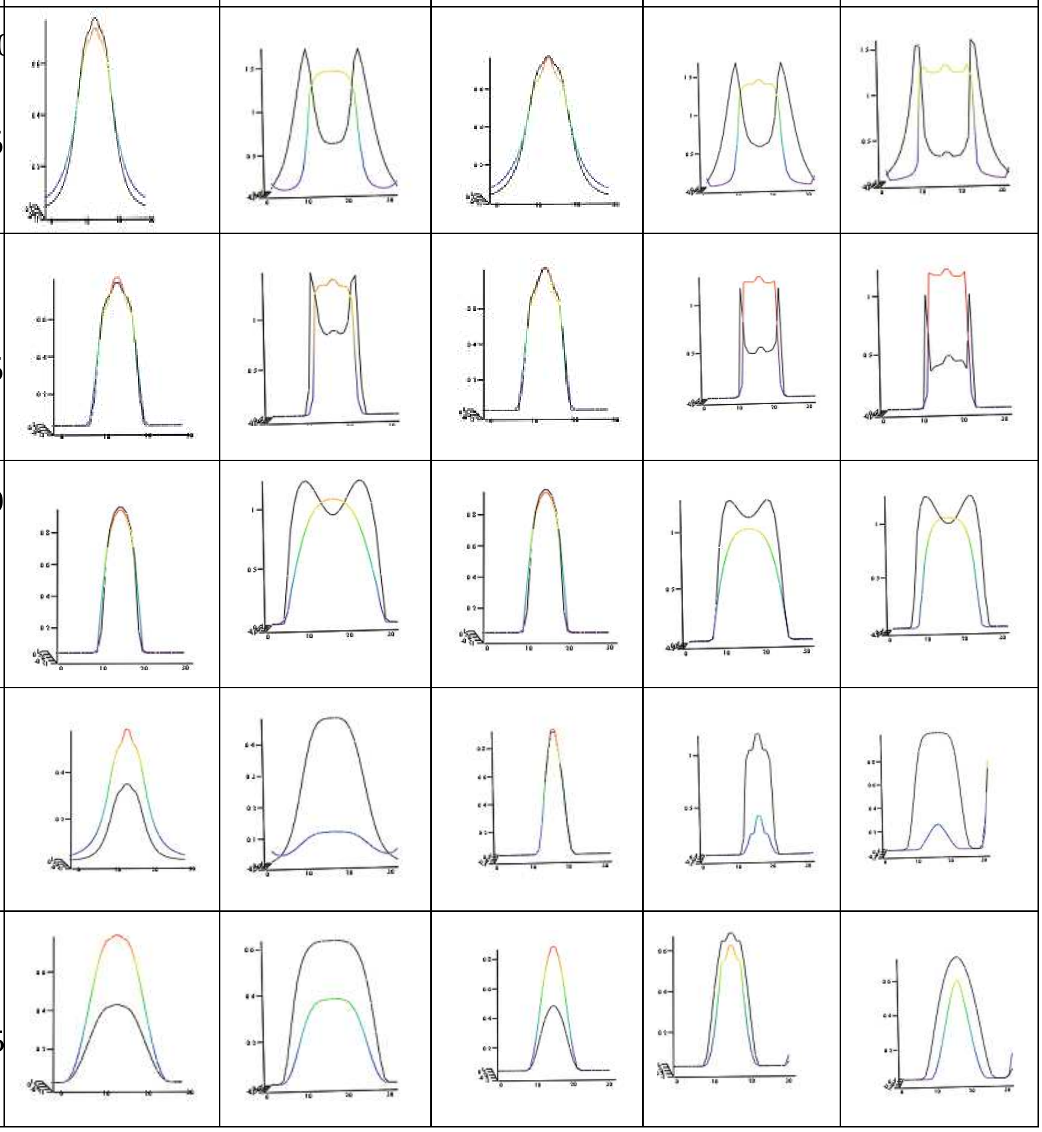

Figure 1: Results of Numerical Experiment

REFERENCES

1. Ахромеева ТСи др. *Нестационарные структуры и диффузионный хаос* (М.: Наука, 1992)
2. de1-Casti11o-Negrete D, Carreras B A Phys. Plasmas 9 118 (2002)
3. del-Castillo-Negrete D, Саггe B A, Lynch V Physica D 168-169 45 (2002)
4. Jorne J L Theor. Biol. 55 529 (1975)
5. A1mirantis Y, Papageorgiou S L Theor. Biol. 151 289 (1991)
6. Иваницкий Г Р, Медвинский А Б, Цыганов М А *УФН*161 (4) 13 (1991)
7. Wu Y, Zhao X Physica D 200 325 (2005)
8. Kuznetsov Yu A et a1. L Math. Biol. 32 219 (1994)
9. Кузнецов Ю А и др. "Кросс-диффузионная модель динамики границы леса", в сб. *Проблемы экологического мониторинга и моделирования экосистем* Т. 26 (СПб.: Гидрометиздат, 1996) с. 213
10. Burridge R, KnopoffL Вм//. Seismol. Soc. Am. 57 341 (1967)
11. Cartwright J HE, Hernandez-Garcia E, Piro O Phys. Pev. Lett. 79 527(1997)
12. Stuart A M /MA L Math. *Appl.* Med. Biol. 10 149 (1993)
13. Murray J.D. Mathematical Biology. I. An Introduction (Third Edition). – N.Y., Berlin, Heidelberg: Springer Verlag, 2001. – 551 p.,
14. M. Aripov (1997). «Approximate Self-similar Approach tu Solve Quasilinear Parabolic Equation» Experimentation, Modeling and Computation in Flow Turbulence and Combustion vol. 2. p. 19- 26.
15. Арипов М. Метод эталонных уравнений для решения нелинейных краевых задач Ташкент, Фан, 1988, 137 с.
16. Белотелов Н.В., Лобанов А.И.Популяционные модели с нелинейной диффузией. // Математическое моделирование. –М.; 1997, №12, стр. 43-56.
17. В. Вольтерра. Математическая теория борьбы за существование -М.: Наука, 1976, 288 с.
18. Гаузе Г.Ф. О процессах уничтожения одного вида другим в популяциях инфузорий // Зоологический журнал,1934, т.13, №1.
19. Aripov M., Muhammadiev J. Asymptotic behaviour of automodel solutions for one system of quasilinear equations of parabolic type. Buletin Stiintific-Universitatea din Pitesti, Seria Matematica si Informatica. N 3. 1999. pg. 19-40
20. Aripov M.M. Muhamediyeva D.K. To the numerical modeling of self-similar solutions of reaction-diffusion system of the one task of biological population of Kolmogorov-Fisher type. International Journal of Engineering and Technology. Vol-02, Iss-11, Nov-2013. India. 2013.

21. Арипов М.М. Мухамедиева Д.К. Подходы к решению одной задачи биологической популяции. Вопросы вычислительной и прикладной математики. -Ташкент. 2013. Вып.129. -С.22-31.

22. Мари Дж. Нелинейные диффузионные уравнения в биологии. М., Мир,1983, 397 стр.

23. Колмогоров А. Н., Петровский И. Г., Пискунов Н.С.Исследование уравнения диффузии, соединенной с возрастанием количества вещества и его применение к одной биологической проблеме. Бюллетень МГУ, сер. Математика и механика, т. 1, 1-25.